Flexibility premium of emissions permits

Citation for published version:
Taschini, L 2020, 'Flexibility premium of emissions permits', *Journal of Economic Dynamics and Control*, pp. 1-15. https://doi.org/10.1016/j.jedc.2020.104013

Digital Object Identifier (DOI):
10.1016/j.jedc.2020.104013

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Publisher's PDF, also known as Version of record

Published In:
Journal of Economic Dynamics and Control

Journal of Economic Dynamics & Control xxx (xxxx) xxx

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Flexibility premium of emissions permits☆

Luca Taschini[a,b,c,*,*]

[a] University of Edinburgh Business School, United Kingdom
[b] Grantham Research Institute (LSE), United Kingdom
[c] DSE - University of Verona, Italy

ARTICLE INFO

Article history:
Received 24 October 2019
Revised 17 September 2020
Accepted 8 October 2020
Available online xxx

JEL classification:
Q50
Q52
C61
D81
G13

Keywords:
Carbon markets
Climate change
Cap-and-trade
Implementation delay
Irreversible investment

ABSTRACT

Investments in abatement technology are often characterised by irreversibility and significant implementation lags, whereas emissions permits can be traded at any time. As such, abatement and emissions permit trading systems are hardly perfect substitutes. We formally study the flexibility of emissions permits and propose a unified framework to rationalise the impact of both investment/divestment lags and irreversibility in relation to the price of emissions permits. Using option pricing concepts, we reformulate the technology adoption problem in terms of the technology's characteristics (irreversibility and implementation lags) and offer a conceptual quantification of the flexibility premium of emissions permits.

1. Introduction

A cap-and-trade programme sets a maximum level of pollution – a cap – and distributes emissions permits among businesses that produce emissions. Currently, jurisdictions comprising 42% of global GDP have adopted (or are in the process of adopting) cap-and-trade programmes, covering approximately 15% of global greenhouse gas emissions. Their global coverage is growing as a key policy lever on the road to net-zero emissions (ICAP, 2020). The chief objective of a cap-and-trade programme is to curb emissions by encouraging businesses to adopt low-polluting technologies and to innovate, ultimately becoming more efficient (European Commission, 2005). However, the literature acknowledges that currently adopted abatement technologies are mature, low-investment, ready to implement and easily reversible.[1] In other words, there is

☆ The author gratefully acknowledges funding from the Grantham Foundation for the Protection of the Environment, the UK Economic and Social Research Council (ESRC) through the Centre for Climate Change Economics and Policy, and the Climate Economics Chair. We received helpful comments from participants at the Bachelier, CESifo, CMS, EAERE and INREC annual conferences and seminar participants at Kings College London, LSE, Essen-Duisburg, Ritsumeikan, University of Tokyo, University of Zurich. The authors would like to thank Jiro Akahori, Federica Buricco, Luciano Campi, René Carmona, David Mauer, Yoichi Otsubo and Gianangelo Taschini for valuable insights and conversations at different stages of this project. The usual disclaimers apply.

* Correspondence to: University of Edinburgh Business School 29 Buccleuch Place, Edinburgh, EH8 9JS United Kingdom.
E-mail addresses: luca.taschini@ed.ac.uk, l.taschini1@lse.ac.uk

[1] Most current cap-and-trade programmes have primarily encouraged fuel-switching – switching from cheap-but-dirty coal to expensive-but-cleaner gas (Goulder, 2013, Schmalensee and Stavins, 2017).

https://doi.org/10.1016/j.jedc.2020.104013
0165-1889/© 2020 Published by Elsevier B.V.

currently no empirical illustration of the alternative scenario, where firms adopt innovative abatement technologies that require high upfront investments, have relatively long implementation lags and are difficult or very costly to reverse. A recent roadmap for decarbonisation suggests that low-carbon investment needs to increase by an order of magnitude by 2030 (Rockstróm et al., 2017). Will cap-and-trade regulations spur substantial investments in advanced abatement technologies? This paper proposes a general model to describe and rigorously analyse how firms respond to cap-and-trade regulations when abatement technologies are characterised by implementation lags and irreversibility. It provides new insights for the role of permit markets in achieving net-zero emissions, by explicitly considering the characteristics of alternative abatement technologies, and contributes to the debate about which carbon price levels are consistent with climate neutrality (Stern and Stiglitz, 2017 and Burke et al., 2019).

Empirical research has found broad support for the adoption of mature abatement technologies. The Acid Rain Program, a major cap-and-trade programme to control SO_2 emissions from US coal plants, encouraged firms to adopt readily available flue-gas desulphurisation technologies – so-called scrubbers (Calel, 2020). Besides accommodating scrubbers, plants also switched to lower-sulphur coal, a carbon management practice that is relatively easy to implement and that does not require upfront capital investments (Schmalensee et al., 1998, Cicala, 2015). The NO_X Budget Trading Program spurred the adoption of straightforward add-on control technologies, such as selective catalytic reduction (Fowlie et al., 2012). The Environmental Protection Agency's market-based phase-down of leaded gasoline promoted the adoption of a relatively inexpensive and well-known substitute technology, pentane-hexane isomerisation (Kerr and Newell, 2003, Calel, 2020). Similarly, the European Union Emissions Trading System (EU ETS), the first large-scale cap-and-trade programme for carbon emissions, has largely encouraged low-investment cost-management strategies, such as fuel-switching. Studies have suggested that a large proportion of the emissions reductions under the EU ETS can be accounted for by fuel-switching in the electricity generation sector (Delarue et al., 2008, Carmona et al., 2009, Delarue et al., 2010). By switching from dirty coal-fired generation to more efficient gas-fired output, electricity generators improve fuel-mix efficiency and reduce the carbon-intensity of their output. This growing body of empirical work highlights a story of adopting mature technologies that can be implemented *soon after* the decision to invest and where the investment strategy *can be reversed*, for example by switching back from lower-sulphur coal to higher-sulphur coal generation. These technologies, however, are not sufficient to deliver the transformation required to achieve net-zero emissions.

Less mature technologies tend to require larger upfront capital investments, and new equipment is durable and often specialised. In other words, these capital investments are *irreversible*. Chao and Wilson (1993) are among the first to recognise that, because of the risk presented by irreversible commitments, abatement technologies are not perfect substitutes for emissions permits in the context of cap-and-trade regulations. Emissions permit purchasing can be adapted to changing market conditions, whereas a specific abatement technology might be under-utilised if product demand falls. Focusing on the SO_2 market in the US, Chao and Wilson (1993) consider the case where an SO_2 scrubber can be installed *instantly* at any time, and at a constant marginal cost. Assuming future product demand is known, Chao and Wilson show that, in equilibrium, emissions permit prices equal the marginal cost of the scrubber. This is consistent with the results of the emissions-constrained problem originally formulated by Montgomery (1972) under certainty and extended by Cronshaw and Kruse (1996), Rubin (1996), Schennach (2000) and, more recently, Carmona et al. (2009), Kollenberg and Taschini (2016), and Hitzemann and Uhrig-Homburg (2018). In practice, however, product demand varies in response to fluctuations in the input and output product markets. Thus, under a more realistic framework where demand is uncertain, Chao and Wilson (1993) maintain that scrubbers become imperfect substitutes for emissions permits. The rationale is quite intuitive: scrubbers are irreversible and durable investments, whereas emissions permits can be purchased as needed in response to changes in product demand. Consequently, under uncertainty, the price of emissions permits is expected to exceed the marginal cost of a scrubber by an amount that corresponds to what Chao and Wilson dubbed the 'option value', or the *reversibility* optionality value of emissions permits. Effectively, the reversibility optionality represents the ability to *divest*, namely to liquidate or sell the technology.

By the same token, reversible abatement technologies that can be *instantly* installed would be a perfect substitute for emissions permits. In practice, however, decisions to install even reversible abatement technologies are typically characterised by significant *implementation lags*. With a few exceptions, off-the-shelf mature abatement technologies are not immediately brought online following the decision to invest (Bar-Ilan and Strange, 1996). Implementation lags might have profound impacts on the economic viability of the abatement decision undertaken. In fact, depending on the evolution of the product market, the decision to invest in, or divest from, abatement (in the case of reversible strategies) might have partially lost its economic attractiveness *during* the implementation lag. Thus, even *reversible* but *non-instantaneous* abatement investments are inferior substitutes for emissions permits. Crucially, abatement investments might become too costly if product demand falls during the implementation period, whereas emissions permit trades can be easily adjusted to market conditions.[2] Consequently, the total price embeds both the *divestment*, or *reversibility*, and the *immediateness* optionalities. Together they constitute a *flexibility premium*, which is embedded in the price of emissions permits. To sum up, a firm that is subject to cap-and-trade is only indifferent between investing in abatement and trading emissions permits when each strategy incurs the same total costs, results in the same amount of emissions reduction and when, simultaneously, (i) technology *divestment* is possible and (ii) the abatement investment is *instantaneous*.

[2] Most cap-and-trade programmes are characterised by a liquid market for emissions permits and virtually zero or negligible transaction costs.

Following Brennan and Schwartz (1985) and McDonald and Siegel (1986) the basic formulation of the investment problem under uncertainty has been extended in many different directions to account for the reversibility of the investment and implementation delays. Dixit (1989) uses the real options approach to examine entry and exit from a productive activity. Using dynamic programming, Brekke and Øksendal (1994) examine a resource extraction problem and determine the optimal opening and closing times. Investment lags have been incorporated into the irreversible investment framework by Majd and Pindyck (1987), Bar-Ilan and Strange (1996) and Zhou (2000), where the investment lag, called time-to-build in this literature, captures the time between the instant the decision to invest is taken and the instant the investment decision is implemented and starts generating new revenues.[3] Under the time-to-build scenario, the investment decision is taken at a specific point in time when the optimal investment threshold is reached, but the implementation occurs only after some time (the lag). The decision to invest might have lost its economic attractiveness by then. But in reality, a firm's investment decision incorporates their expectations about market conditions at the end of the implementation lag. They may reserve the right to withdraw the decision to invest at any time during the implementation lag, if the investment loses its attractiveness, or they may delay the decision to invest until further information is accumulated. For example, construction lags affect decisions around residential construction projects (Oh and Yoon, 2020) and shipbuilding (Kalouptsidi, 2014).[4] In other words, the decision can be said to occur *over a period of time*, and depends on the economic attractiveness of that decision remaining positive over the course of that period. This scenario is effectively represented by the Parisian constraint[5] which incorporates both the attractiveness element and the time span over which it must be maintained. It is used to select an optimal dividend policy in the corporate finance literature (see Dassios and Wu, 2009 and Cheung and Wong, 2017), but here, we simply apply the concept to describe the context within which the decision is taken.

Returning to our setup, Chao and Wilson (1993) consider a cap-and-trade programme in the presence of investment constraints. Abatement constraints are modelled by assuming that the investment is irreversible, i.e. that divestment is not possible. Permits and abatement are no longer perfect substitutes and the relationship between permit prices and marginal abatement costs is distorted. In fact, Chao and Wilson show that the price of emissions permits can exceed the marginal cost of abatement by an amount that captures permits' reversibility/divestment optionality. They also show that investments in abatement are reduced when the uncertainty about product demand is significant. A later study by Zhao (2003) confirms this last result in a general equilibrium framework with stochastic abatement costs. However, both these papers limit their analysis to irreversible and instantaneous investment decisions, a key simplification that allows them to formulate a tractable investment problem.

This paper provides a significant generalisation of a firm's investment problem with cap-and-trade regulations under more realistic assumptions. The economic analysis is significantly enriched as the firm now considers both a (one-time) reversibility optionality and the implementation lags following the abatement investment and divestment decisions. To keep the model tractable, we concentrate on the main considerations discussed earlier: (1) either the technology is too specialised and it is not possible to divest (irreversible investment) or, conversely, it is possible to liquidate the equipment (one-time reversible investment); and (2) the timing of the implementation of the investment (or divestment) decision (length of implementation lags). Despite the binary alternative between irreversible and (one-time) reversible investments, the formulation of the decision problem under the Parisian setup appears rather complex. Yet, we are still able to provide a remarkably tractable analysis of this problem using an intuitive decomposition of the permit price based on the flexibility premium. The decomposition and quantification of the flexibility premium represent an important extension to the existing literature, which fails to take these characteristics into account. It also offer important insights by reflecting the real-world trade-offs involved in existing emissions abatement options.

The remainder of the paper is organised as follows. Section 2 proposes an intuitive decomposition of the permit price. The model and the main results about the flexibility premium are presented in Section 3. Section 4 provides a quantitative illustration of the flexibility premium. Section 5 concludes.

2. The decomposition of the permit price

Consider an economy subject to cap-and-trade regulations which impose a cap on cumulative emissions over a prespecified period, also called the programme phase. To achieve compliance and remain within the emissions limit, regulated firms can reduce their emissions by adopting an emissions abatement technology. Alternatively, firms can purchase the number of permits required to offset their emissions in excess of their authorised limit. Within the canonical microeconomic framework (Rubin, 1996, Schennach, 2000, Kollenberg and Taschini, 2016 and Lintunen and Kuusela, 2018) and experimental framework (Stranlund et al., 2011 and references therein), these two compliance strategies are assumed to be perfect substitutes, insofar as (i) they each generate the same total reduction in emissions at the same total cost, and, simultaneously

[3] With an application to portfolio divestment and resource extraction, Øksendal (2005) studies the effect of delayed information and identifies optimal divestment rules. The combination of investment reversibility and implementation delays has found numerous applications in economics and finance (see de Almeida and Zemsky, 2003, Bultan, 2005, Sødal et al., 2008, Paciorek, 2013, Bloom, 2014, Aguerrevere, 2015).

[4] Oh and Yoon (2020) observe that 'forward-looking' homebuilders consider their expected return on investment during the entire house construction process. Similarly, Kalouptsidi (2014) reports that construction lags and expected revenues during construction have a substantial impact on the level of investment in new ship construction.

[5] The Parisian constraint takes its name from the Parisian option – a specific barrier option used in finance. A complete description of the scenario is provided later.

(ii) the abatement investment is reversible and there are no implementation lags. In reality, most technologies are characterised by irreversibility and significant delays between the instant the investment decision is taken and the instant when the investment is implemented.

In this paper, we depart from the assumption of perfect substitutes and examine the implications of a one-time investment reversal (divestment) and implementation lags, thereby extending the work done by Chao and Wilson (1993) and Zhao (2003) who only consider the irreversible and instantaneous case. A one-time reversal is a simplified stand-in for the infinitely reversible case (such as fuel-switching decisions that can be reversed an infinite number of times) but it allows us to concentrate on the contrast between reversible vs non-reversible investments. Furthermore, it is important to incorporate implementation lags since they affect investment and divestment timing as well as the investment value. Since emissions permit trading is both reversible and instantaneous, in contrast with most abatement technology, this work argues that the price of emissions permits should incorporate an 'option value' that reflects this flexibility. We dub this option value the *flexibility premium*.

Below, we propose an intuitive representation of the flexibility premium. First, we determine the permit price that makes a firm indifferent between trading and abatement. Then, we evaluate the difference between the permit prices associated with (1) reversible and instantaneous abatement and (2) reversible and non-instantaneous abatement (the full model allow us to consider irreversibility as well). This difference constitutes the flexibility premium and captures both the reversibility and immediateness optionalities. In the next section we concentrate on the flexibility premium and propose a general expression in terms of the ability to divest (reversibility) and implementation lags, i.e. the key parameters that characterise the abatement technology.

Firm's net operating profits We consider a firm with an abatement investment opportunity modelled as in McDonald and Siegel (1986). We assume that the net operating profits $(S_t, t \geq 0)$ are described by a one-dimensional stochastic process which drives the entire economy. As in the real option pricing literature, we assume that S_t follows a geometric Brownian motion

$$\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t, \qquad S_0 = x,$$

with μ and σ constant parameters and $(Z_t, t \geq 0)$ a standard Brownian motion.[6] To keep the model tractable, and to concentrate on the main considerations discussed in the Introduction, namely irreversibility and implementation lags, we assume that the net operating profits pre– and post-technology adoption are described by the same stochastic process. However, the costly adoption of the abatement technology slashes the instantaneous rate of pollution emission α to zero, where $\alpha > 0$.[7] Finally, we consider a zero-emissions cap regulation. These two extreme model assumptions – zero post-investment emissions and zero-emissions regulations – are innocuous. They help to make our point more vividly, but these values could equally be set to any non-zero level within the same model without any loss of generality.

By convention, the firm's value V_t is defined as the expected sum of the present value of future net operating profits:

$$V_t = \mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(u-t)} S_u du\right],$$

where ρ is the constant discount rate. Our model specification allows us to straightforwardly obtain the dynamics of the firm's value. In fact, it is simple to verify that V_t satisfies the following stochastic differential equation:

$$\frac{dV_t}{V_t} = \mu dt + \sigma dZ_t, \qquad V_0 = \frac{S_0}{\rho - \mu}.$$

Comparing compliance strategies There are two basic compliance strategies in a cap-and-trade programme, sometimes used in combination: abating and trading. Let us consider a programme phase $[0, T]$ and two possible compliance alternatives at time $t = 0$. In the first scenario, the firm decides to undertake no technology adoption and acquire the exact number of permits required to stay within the administered cap at the end of the regulated phase T. Analytically, we can describe this compliance strategy as:

$$V_0 - e^{-\rho T}(\alpha T)P_T,$$

where P_T is the (futures) price of emissions permits at time T. In the second scenario, we consider the case where the regulated firm decides to invest at time s and to divest at time s', with $(0 < s < s' < T)$. We momentarily assume that these decisions are the optimal outcome of an investment–divestment problem and let $F^d(V)$ be the solution of this problem when the abatement technology is reversible and characterised by a pair of *implementation lags* $\{d_I \geq 0, d_D \geq 0\}$ respectively and with $d = (d_I, d_D)$. In particular, let $F^0(V)$ be the solution of the technology adoption problem when the abatement technology is reversible and *instantaneous*. In the next section we provide the complete formulation of the investment–divestment problem in terms of the characteristics of the available abatement technology. Returning to the second scenario, upon investment, the instantaneous rate of emissions is slashed to zero; upon divestment, the instantaneous rate of emissions goes

[6] By convention, the Brownian motion $(Z_t, t \geq 0)$ is defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$.

[7] At the cost of introducing yet more notation, the analysis can be extended to the case where, post-technology adoption, (i) the parameters of the geometric Brownian motion describing the net operating profits differ, and (ii) the new instantaneous emission rate is $\tilde{\alpha} \in [0, \alpha]$. In both situations, the final decomposition of the permit price remains unaltered. In particular, under (ii), there is a change in the total amount of emissions reduction.

back to α. Therefore, there are αs emissions in period $[0, s]$, zero emissions in period $[s, s']$ and $\alpha(T - s')$ emissions in period $[s', T]$. The cap-and-trade regulations require a 100% reduction in emissions for the entire regulated period $[0, T]$. Therefore, at time T, the firm acquires the number of permits required to offset the total amount of emissions. Using the new notation, we can analytically describe this alternative compliance strategy as

$$F^d(V_0) - e^{-\rho T}\alpha(T - s' + s)P_T,$$

where $F^d(V_0)$ is the value of firm investing at time s and divesting at time s'. For the sake of simplicity, this expression neglects the net operating profits generated between $[0, s]$ and $[s', T]$. This approximation does not affect the final decomposition of the permit price and, more importantly, it is irrelevant for the flexibility premium.

These two compliance strategies attain the same total amount of emissions reduction and

$$P_T = e^{\rho T}\frac{V_0 - F^d(V_0)}{\alpha(s' - s)}$$

is the permit price that makes total compliance costs equal. Yet, a firm is indifferent between the two alternative strategies only when trading and abating are perfect substitutes. Here $F^d(V)$ can represent the entire spectrum of possible abatement technologies: from technologies that are reversible and instantaneous to technologies that are irreversible and non-instantaneous. Thus, we can rewrite the (futures) permit price at time $t = 0$ as the combination of two components:

$$P_T = e^{\rho T}\frac{V_0 - F^d(V_0)}{\alpha T} = e^{\rho T}\frac{V_0 - F^0(V_0)}{\alpha\left(s' - s\right)} + e^{\rho T}\overbrace{\frac{F^0(V_0) - F^d(V_0)}{\alpha\left(s' - s\right)}}^{flexibility\ \ premium}. \tag{1}$$

We now exclusively concentrate on the second component. We exclusively consider the technology adoption decision problem and formulate it as an investment and divestment decision problem under a decision constraint that appropriately reflects irreversibility and implementation lags. This allows us to go beyond the alternative decomposition of the permit price and offer a conceptual quantification of the flexibility premium.

3. The flexibility premium

Eq. (1) offers a compact and intuitive interpretation of the flexibility premium in terms of the difference between the solutions of the investment decision problem under two extreme abatement technology scenarios: (1) reversible and instantaneous vs (2) reversible/irreversible and non-instantaneous.[8] However, it remains unclear prima facie how each of these constituents (irreversibility and implementation lags) affect the magnitude of the flexibility premium. To illuminate this further, first we formulate a general investment–divestment problem under the associated setup that appropriately captures the reversibility/divestment optionality and implementation delays. Second, we unpack the flexibility premium in Eq. (1) and describe the cumulative effects of irreversibility and implementation lags on the premium.

We introduce the specification for investment (divestment) costs and implementation delays as follows. If the firm decides to invest (divest), it incurs an upfront fixed cost C_I (C_D). A positive value for C_D indicates a positive liquidation value of the technology: there is a second-hand market or, equivalently, its scrap value is non-zero. A negative C_D indicates that divestment is costly, e.g. the firm might bear expenses associated with plant decommissioning. Furthermore, the firm can face investment and divestment lags; d_I and d_D, respectively. Such lags correspond to the delays between the moment the decision to invest or divest is taken and the instant when the investment or divestment is implemented. The proposed model is sufficiently general to span the entire range of possible implementation lags. At one extreme, there are mature technologies where adoption is practically instantaneous and implementation requires very little time. In this case the lag is virtually zero. The only example of this in reality applies to some electricity plants that are equipped to switch from cheap-but-dirty coal to expensive-but-cleaner gas within a matter of hours. At the other extreme, there are less mature technologies that require large upfront investments in durable equipment whose implementation/installation can take a very long time. The adoption of these technologies might involve multiple investment stages and occasional bottlenecks that further delay the completion of the investment. For example, the adoption of carbon capture – a technology frequently discussed for the decarbonisation of energy-intensive industries like cement, oil refineries, and iron and steel – currently requires several years. At the same time, most of these investments in durable and expensive equipment are completely irreversible. The adoption of SO_2 scrubbers is a striking example. As discussed in Cicala (2015), scrubbers are enormously expensive pieces of equipment that have no second-hand market and are more costly to dismantle than to assemble. By considering finite ($d_D < \infty$) and infinitely long ($d_D \to \infty$) divestment lags, we account for both reversible and irreversible investments, respectively. Basically the model allows us to consider varying lengths of implementation lags $\{d_I, d_D\} \in [0, \infty]$ and a binary alternative between reversible and irreversible investments. When the divestment lag is infinitely long, reversibility is not possible.

The technology adoption problem is formulated as an investment–divestment decision problem. To handle both irreversibility and implementation lags consistent with the empirical evidence and the abatement technology characteristics

[8] The model allows us to consider a binary alternative between reversible and irreversible investments and varying lengths of implementation lags.

described in the Introduction, we borrow the Parisian setup for pricing a particular kind of barrier option.[9] The Parisian condition requires that investment (or divestment) only occurs when the net operating profits post-technology adoption cross a specific threshold and remain there for a specific period of time. This period corresponds to the implementation lags d_I and d_D introduced earlier, whereas the specific threshold levels correspond to the optimal investment and divestment thresholds. According to the Parisian condition, the firm's commitment to invest (divest) depends on the expectation that the market conditions will remain, on average, favourable (unfavourable) once the investment (divestment) decision is implemented (at the end of the implementation lag). In the main text we reformulate the investment–divestment decision problem under the Parisian setup and propose a formula for the flexibility premium.

As in the real options literature, the value of the firm can be written as the expected sum of the investment and divestment abatement opportunities. Given the investment and divestment costs, the firm maximises the present value of its opportunities:

$$F^d(V_0) = \max_{\tau_I<\tau_D} \mathbb{E}_0\left[e^{-\rho\tau_I}(V_{\tau_I}-C_I)\ 1_{\{\tau_I<\infty\}}\ +\ e^{-\rho\tau_D}(C_D-V_{\tau_D})\ 1_{\{\tau_D<\infty\}}\right], \tag{2}$$

where τ_I and τ_D represent the first instants when the firm's value has consecutively spent a given amount of time above or, respectively, below a specific threshold. In other words, the Parisian condition requires that investment or divestment only occurs when the value of the abatement investment crosses specific investment or divestment thresholds h_I or h_D, respectively, and remains there for time period τ_I or τ_D, respectively. The investment τ_I and divestment τ_D stopping times which satisfy the Parisian condition are:

$$\tau_I = \inf\{t \geq 0 : t - g_t^{V_0,h_I} \geq d_I, V_t \geq h_I \mid V_0 = v\},$$
$$\tau_D = \inf\{t \geq \tau_I : t - g_t^{V_0,h_D} \geq d_D, V_t \leq h_D \mid V_0 = v\},$$

where $g_t^{V_0,a}(V)$ represents the last time the process $(V_t, t \geq 0)$ crossed the generic threshold a and formally it is defined as $g_t^{V_0,a}(V) = \sup\{s : s \leq t, V_t = a \mid V_0 = v\}$. For a review of key definitions and the properties of the Parisian condition, we refer readers to the technical appendices (Appendix A and B).

Because we consider the firm to exist in perpetuity, the investment or divestment decision will occur at the first instant when $(V_t, t \geq 0)$ hits some constant optimal threshold h_I^* or, respectively, h_D^*. Thus, with τ_I and τ_D stopping times corresponding to the Parisian condition, we can rewrite the abatement technology adoption problem in terms of optimal investment and divestment thresholds:

$$F^d(V_0) = \max_{h_D\leq h_I,\ V_0\leq h_I} \mathbb{E}_0\left[e^{-\rho\tau_I}(V_{\tau_I}-C_I)\ 1_{\{\tau_I<\infty\}}\ +\ e^{-\rho\tau_D}(C_D-V_{\tau_D})\ 1_{\{\tau_D<\infty\}}\right], \tag{3}$$

We take advantage of some standard results in Revuz and Yor (1991) and Chesney et al. (1997b) and utilise the key results in Costeniuc et al. (2008) to reformulate the investment–divestment decision problem (3). The Appendix contains the key definitions and reproduces the main results in Costeniuc et al. (2008) required to rewrite the problem (3) in terms of the key model parameters (the implementation lags and the investment and divestment costs) and the main decision variables (the investment and divestment thresholds). Below we report the final reformulation of the investment–divestment problem:[10]

$$\begin{aligned} F^d(V_0) = \max_{h_D\leq h_I,\ V_0\leq h_I} & \left(\frac{V_0}{h_I}\right)^{\psi_1} \frac{\phi(b\sqrt{d_I})}{\phi(\sqrt{(2\rho+b^2)d_I})} \Bigg\{ h_I \frac{\phi(\sqrt{d_I}(\sigma+b))}{\phi(b\sqrt{d_I})} - C_I \\ & + \left(\frac{h_I}{h_D}\right)^{\psi_2} \frac{\phi(-\sqrt{(2\rho+b^2)d_I})}{\phi(\sqrt{(2\rho+b^2)d_D})} \frac{\phi(-b\sqrt{d_D})}{\phi(b\sqrt{d_I})} \left(C_D - h_D \frac{\phi(-(b+\sigma)\sqrt{d_D})}{\phi(-b\sqrt{d_D})}\right) \Bigg\} \end{aligned} \tag{4}$$

where, following the literature on Parisian options, the original dynamics of the firm's net operating profits post-technology adoption are translated in terms of the drifted Brownian motion

$$V_t = V_0 e^{\sigma X_t}, \quad \text{where } X_t = bt + Z_t, \quad \text{and} \quad b = \frac{\mu - \frac{\sigma^2}{2}}{\sigma}. \tag{5}$$

To simplify for the sake of legibility, we adopt the following notations:

$$\psi_1 = \frac{-b+\sqrt{2\rho+b^2}}{\sigma} \quad \text{and} \quad \psi_2 = \frac{-b-\sqrt{2\rho+b^2}}{\sigma}.$$

And ϕ is defined as:

$$\phi(z) = \mathbb{E}(\exp(zx)) = \int_0^\infty \exp(zx)\mathbb{P}_x(dx).$$

[9] The Parisian option is a barrier option in option pricing theory. It specifies that barrier activation and deactivation is conditional on the underlying price staying beyond the barrier for a pre-determined consecutive period of time, Chesney et al. (1997b).

[10] For a detailed derivation of the problem reformulation, we refer interested readers to the technical appendix in Costeniuc et al. (2008).

Costeniuc et al. (2008) offer a formal and rigorous analysis of the problem (4). Below we reproduce the key expressions of interest and present an analytical description of the flexibility premium in terms of the solutions of the abatement technology adoption problem under the two abatement technology scenarios: (1) reversible and instantaneous vs (2) reversible/irreversible and non-instantaneous.

First, we take the partial derivative with respect to h_D and, solving for the critical value, we obtain the optimal divestment threshold

$$h_D^* = \frac{\phi(-b\sqrt{d_D})}{\phi(-(b+\sigma)\sqrt{d_D})}\frac{\psi_2 C_D}{\psi_2 - 1}. \tag{6}$$

When $d_D = 0$ we reproduce the expression for the optimal *instantaneous* divestment threshold $h_{ND}^* = \frac{\psi_2 C_D}{\psi_2 - 1}$, which is the result obtained by Dixit and Pindyck (1994). Noting that ϕ is an increasing function, it is straightforward to show that $h_{ND}^* \leq h_D^*$ when $b > 0$: divestment lags would lead one to anticipate divestment, as argued by Bar-Ilan and Strange (1996) and Sødal (2006). Increasing the second-hand market value of the abatement technology, $C_D > 0$, may even speed up divestment if the liquidation of the abatement investment outstrips the sum of future net profits.

Second, we take the partial derivative with respect to h_I and, solving for the critical value, we obtain the optimal investment threshold h_I^* that corresponds to the solution of $h_I^* = \max\{V_0, x^*\}$ where x^* solves the implicit equation

$$x = \frac{\psi_1 C_I}{\psi_1 - 1}\frac{\phi(b\sqrt{d_I})}{\phi((b+\sigma)\sqrt{d_I})} + \left(\frac{x}{h_D^*}\right)^{\psi_2}\frac{\psi_2 - \psi_1}{\psi_1 - 1}\frac{\phi(-\sqrt{(2\rho + b^2)d_I})}{\phi(\sqrt{(2\rho + b^2)d_D})}\frac{\phi(-b\sqrt{d_D})}{\phi((b+\sigma)\sqrt{d_I})}\frac{C_D}{1-\psi_2}. \tag{7}$$

When both $d_I = 0$ and $d_D = 0$, we obtain the optimal *instantaneous* investment threshold h_{NI}^*, echoing Dixit and Pindyck's result for the case of an instantaneous and reversible investment problem. In particular, h_{NI}^* solves the implicit equation

$$x = \frac{\psi_1 C_I}{\psi_1 - 1} + \left(\frac{x}{h_{ND}^*}\right)^{\psi_2}\frac{\psi_2 - \psi_1}{\psi_1 - 1}\frac{C_D}{1-\psi_2}.$$

We now have all the necessary results to analytically describe the effects of irreversibility and implementation lags on the flexibility premium component in Eq. (1). We can solve the optimal investment–divestment problem under the Parisian setup and finally replace the generic expression of the flexibility premium with the optimal solutions of the problem when the investment is (1) reversible and instantaneous, $F^{0^*}(V_0)$, and (2a) reversible and non-instantaneous or (2b) irreversible and non-instantaneous, $F^{d^*}(V_0)$. Again, the binary alternative of reversible vs non-reversible is attainable when the divestment lag is infinitely long. Let θ be the flexibility premium normalised for the number of permits required to offset the total amount of emissions

$$\theta = \frac{F^{0^*}(V_0) - F^{d^*}(V_0)}{\alpha T}, \tag{8}$$

where the investment and divestment lags are $d_I = 0$ and $d_D = 0$ for $F^0(V_0)$ and $d_D > 0$ and $d_I > 0$ for $F^d(V_0)$.

The expression for θ offers a conceptual description of the flexibility premium that describes how firms respond to cap-and-trade regulations when abatement technologies have relatively long implementation lags and are difficult and costly to reverse. Now that we have a unified framework to model the characteristics of the available abatement technology, the next section illustrates how to quantify the impact of investment lags and irreversibility on the price of emissions permits.

4. Quantitative illustration

In this section we employ our theoretical results to illustrate how implementation lags and irreversibility affect the magnitude of the flexibility premium, extending the results in Chao and Wilson (1993) which are limited to the irreversible and instantaneous abatement case. To that end, we perform a numerical evaluation of the investment–divestment decision problem where the Parisian condition simultaneously handles the timing of the implementation of the investment (instantaneous vs non-instantaneous) and the ability to divest (reversible vs irreversible).

Illustration settings We first introduce the general setting of the numerical example. The firm's value at $t = 0$ is set, for normalisation, to a value $V_0 = 100$. The fixed investment cost is $C_I = 150$, implying that the ratio of the current firm's value to fixed cost is 1 to 1.5. Also, we set the divestment value $C_D = 50$, implying the existence of a second-hand market for the abatement technology. Finally, we set the continuously compounded rate of discount $\rho = 0.13$, the drift rate $\mu = 0.05$ and the volatility rate $\sigma = 0.40$. The emission rate pre-technology adoption is $\alpha = 1$ unit of emissions per unit of time (one year, in this example). The cap-and-trade regulation specifies zero emissions within 10 years, $T = 10$. Hence, the required cumulative emissions reduction totals 10.

The investment–divestment decision problem under the Parisian condition is sufficiently general to allow us to model the case where the abatement technology is (1) instantaneous and reversible, $d_I = d_D = 0$, (2a) non-instantaneous and reversible, $d_I > 0$ and $d_D > 0$, and, finally, (2b) non-instantaneous and irreversible, $d_I > 0$ and $d_D \to \infty$. We first illustrate (1) and (2a) considering different combinations of investment and divestment lags. Then, we discuss the effects on the flexibility premium of varying investment and divestment costs and different values of the volatility of the net operating profits. This way

Table 1
Flexibility premium when $\rho = 0.13, \mu = 0.05, \sigma = 0.40, C_D = 50; C_I = 150, V_0 = 100, \alpha = 1, T = 10, s = 0$.

d_I/d_D	0	2	4
0	0	0.001	0.002
3	0.268	0.269	0.271
5	0.454	0.456	0.457

Table 2
Flexibility premium when $\rho = 0.13, \mu = 0.05, \sigma = 0.40, C_D = 50, C_I = 170, V_0 = 100, \alpha = 1, T = 10, s = 0$.

d_I/d_D	0	2	4
0	0	0.001	0.002
3	0.252	0.254	0.255
5	0.428	0.429	0.430

Table 3
Flexibility premium when $\rho = 0.13, \mu = 0.05, \sigma = 0.40, C_D = 70, C_I = 150, V_0 = 100, \alpha = 1, T = 10, s = 0$.

d_I/d_D	0	2	4
0	0	0.002	0.005
3	0.262	0.270	0.272
5	0.452	0.455	0.458

we model the main groups of industries currently regulated under cap-and-trade programmes: (1) those few electricity generation industries characterised by virtually instantaneous and reversible abatement options and (2a) those energy-intensive industries characterised by long investment lags or (2b) irreversible investments in durable equipment.

The investment–divestment decision problem is solved considering different pairs of investment–divestment lags: $d_I \in \{0, 3, 5\}$ and $d_D \in \{0, 2, 4\}$. The range of these intervals is sufficient to illustrate the general effect of varying degrees of implementation lags on the flexibility premium. Table 1 reports the values of the flexibility premium for every possible combination. When divestment is possible and both investment and divestment are instantaneous, $(d_I, d_D) = (0, 0)$ and the flexibility premium is zero. At the opposite extreme, when investment and divestment each take approximately half of the length of the regulated phase, $(d_I, d_D) = (5, 4)$, the premium is in excess of 0.45 per permit. To place this number into context, the flexibility premium corresponds to a non-negligible 4.5% of the firm's initial value.[11]

The flexibility premium θ combines the immediateness and reversibility optionality of trading emissions permits. Intuitively, the premium increases when the investment/divestment lag increases, albeit in a non-linear fashion. Importantly, the effect on the premium of an increasing d_D is less pronounced than that of d_I. The reversibility of the abatement investment justifies this result. More importantly, it illustrates that reversibility bears a larger importance in determining the value of the flexibility premium and our theoretical results readily provide a quantification of this effect.

In what follows, we study the sensitivity of the flexibility premium around different values for the investment and divestment costs, and the volatility of net operating profits. The effects of varying emission rates and lengths of the regulated period can be trivially inferred from Eq. (8) and are therefore not discussed here.

Investment and divestment costs The higher the investment cost C_I, the higher the trigger threshold for investment. This holds both in the case of instantaneous and non-instantaneous adoption of the abatement technology. This is the result in Majd and Pindyck (1987) and Dixit (1989) for the case of instantaneous investments and in Bar-Ilan and Strange (1996) for the case of delayed investments. A higher investment threshold corresponds to a lower value of the solution of the investment–divestment problem. Crucially, the effect of an increasing C_I is more pronounced on $F^d(V_0)$ than on $F^0(V_0)$. These findings provide an alternative confirmation to the numerical results in Bar-Ilan and Strange (1996). Table 2 reports the values of the flexibility premium for $C_I = 170$, holding everything else constant. Overall, an increasing C_I coincides with a lower flexibility premium. The intuition behind these results is that, ceteris paribus, a higher investment cost makes the abatement technology adoption less attractive compared to trading permits.

Similarly, the higher the second-hand market value of the abatement technology, the stronger the incentive to liquidate the investment. Table 3 reports the values of the flexibility premium for $C_D = 70$ holding everything else constant. The impact of varying divestment costs is however less clear-cut. An increasing C_D corresponds to a higher flexibility premium

[11] To obtain this figure, first consider the number of permits required to achieve the prescribed zero emission reduction, 10 permits over the 10−year phase. Hence the cumulative flexibility premium is 4.5 for a firm whose initial value is $V_0 = 100$.

Table 4
Optimal instantaneous irreversible investment threshold h^*_{II}, and investment h^*_I and divestment h^*_D threshold. The parameters we used are $\rho = 0.13, \mu = 0.05, C_D = 50, C_I = 150, V_0 = 100, \alpha = 1$.

		$d_I = d_D = 3$		$d_I = d_D = 5$	
σ	h^*_{II}	h^*_I	h^*_D	h^*_I	h^*_D
0.05	249.62	204.89	52.01	185.06	52.44
0.06	252.08	203.29	52.50	183.18	53.14
0.07	254.91	201.75	52.95	181.20	53.84
0.08	258.08	200.31	53.36	179.22	54.52
0.09	261.55	199.00	53.74	177.27	55.18
0.1	265.32	197.80	54.08	175.37	55.83
0.11	269.35	196.72	54.40	173.54	56.46
0.12	273.63	195.73	54.68	171.77	57.07
0.13	278.14	194.84	54.94	170.06	57.67
0.14	282.87	194.02	55.18	168.41	58.25
0.15	287.80	193.28	55.40	166.82	58.82
0.16	292.94	192.60	55.60	165.28	59.39
0.17	298.26	191.97	55.78	163.78	59.94
0.18	303.76	191.38	55.96	162.33	60.49
0.19	309.43	190.84	56.12	160.91	61.03
0.2	315.28	190.34	56.28	159.53	61.57
0.21	321.30	189.87	56.42	158.17	62.11
0.22	327.48	189.43	56.56	156.85	62.65
0.23	333.81	189.01	56.70	155.55	63.18
0.24	340.31	188.62	56.83	154.28	63.72
0.25	346.97	188.25	56.96	153.03	64.26
0.26	353.78	187.90	57.08	151.80	64.80
0.27	360.74	187.57	57.20	150.59	65.34
0.28	367.86	187.25	57.32	149.40	65.89
0.29	375.13	186.95	57.44	148.23	66.44
0.3	382.55	186.66	57.56	147.07	67.00
0.31	390.12	186.38	57.68	145.92	67.57
0.32	397.85	186.11	57.80	144.79	68.14
0.33	405.73	185.84	57.93	143.67	68.72
0.34	413.76	185.58	58.05	142.55	69.30
0.35	421.94	185.32	58.18	141.45	69.89
0.36	430.28	185.07	58.30	140.36	70.50
0.37	438.76	184.82	58.43	139.27	71.11
0.38	447.40	184.57	58.57	138.18	71.73
0.39	456.20	184.32	58.70	137.10	72.36
0.4	465.15	184.06	58.84	136.03	73.00

mainly when the length of the divestment lag exceeds the length of the investment lag ($d_D = 4, d_I = 3,$ flexibility premium $= 0.272$).

Uncertainty about future net operating profits We now consider the effect of uncertainty on the investment–divestment decision problem when the investment is instantaneous and irreversible vs non-instantaneous and reversible.

When $d_I = 0$ and $d_D \to \infty$, the investment threshold becomes

$$h^*_{II} = \frac{\psi_1 C_I}{\psi_1 - 1},$$

where h^*_{II} represents the optimal investment threshold when divestment is not possible and there is no investment lag. As σ increases, the investment threshold h^*_{II} rises. The greater the uncertainty about future market conditions, the stronger the incentive to wait and postpone the decision. If market conditions prove unfavourable, a committed firm will regret the adoption of the abatement technology, while an uncommitted firm can choose to trade permits or invest at a later stage. As reported in Table 4, the benefit of waiting rises with uncertainty because the probability of market conditions bad enough to prompt regret also rises with uncertainty. The opportunity cost of waiting is the foregone income post-technology adoption, which depends on the net operating profits during the postponement. Since the firm can adopt the irreversible technology immediately, the opportunity cost of a postponement is independent of uncertainty. This is consistent with the conventional result in real options literature when investments are irreversible and instantaneous (Pindyck, 1988).

The presence of implementation delays changes the firms choice by making the opportunity cost of waiting contingent on uncertainty and the length of the implementation window. With lags, the foregone income is now uncertain; it depends on the random path followed by net operating profits over the lag period. The firm's choice changes even more when divestment, even at a cost, is possible. Now the returns from the adoption of an abatement technology are bounded below. This means that expected net operating profits increase with uncertainty, and therefore that the opportunity cost of waiting

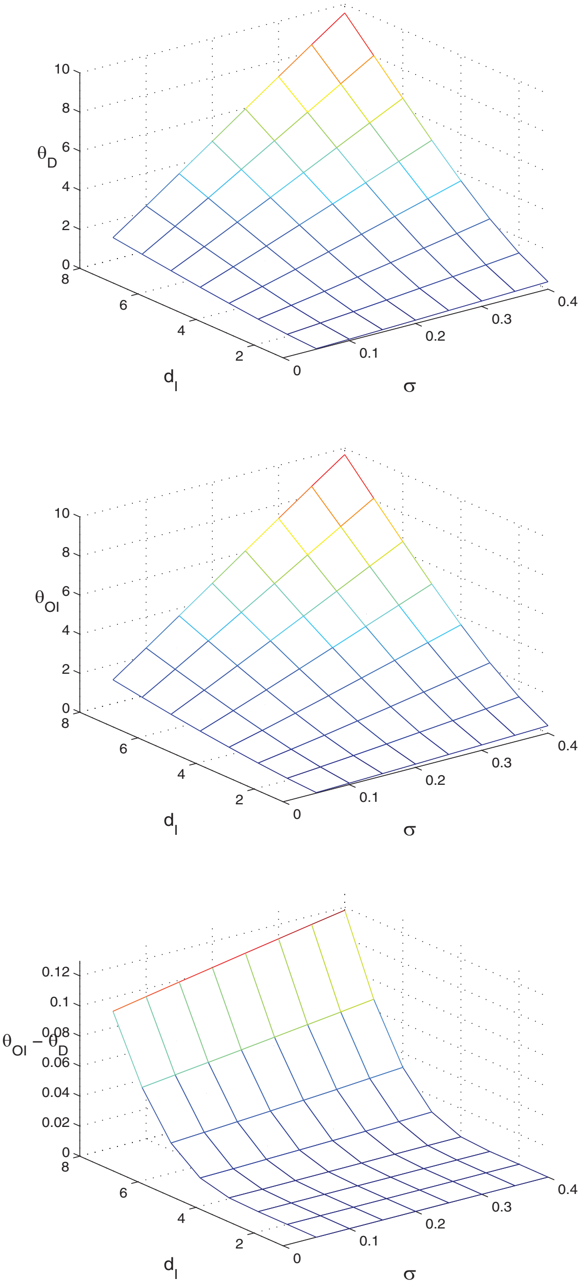

Fig. 1. θ_D and θ_{OI} are, respectively, the flexibility premium when the investment is reversible (top graph) and irreversible (middle graph). The flexibility premia are computed considering different combinations of the uncertainty about future net operating profits, σ, and the investment lag, d_I. The lower graph represents the difference between the two premia. All premia are scaled up to improve the readability of the graphs.

now rises with uncertainty. Thus, when the investment is non-instantaneous and reversible, uncertainty may hasten investment and divestment decisions. As reported in Table 4, h_I^* falls from 204.89 to 184.06 and h_D^* climbs from 52.01 to 58.84 when $d_I = d_D = 3$. The longer the investment/divestment lags, the stronger the effect. Therefore, by extending the model to realistically capture both reversibility and implementation lags, the conventional findings about the effect of uncertainty on investments are reversed. This confirms the numerical results in Bar-Ilan and Strange (1996)and Sødal (2006).

Fig. 1 graphically illustrates the combined effect of implementation lags and volatility on the flexibility premium. The larger σ and d_I, the larger the flexibility premium. These analysis of the effect of different values of the volatility of net operating profits provide an alternative qualification and extension of the results in Chao and Wilson (1993) and Zhao (2003), who studied the price of permits but limited their analysis exclusively to the case of irreversible and instantaneous abatement technologies.

Irreversible investment Finally, we illustrate the non-instantaneous and irreversible abatement investment case. By letting $d_D \to \infty$, we offer a general characterisation and readily quantify the 'option value' in Chao and Wilson (1993), who were the first to study the reversibility optionality embedded in the total price of emissions permits but fall short to consider the immediateness optionality. When $d_D \to \infty$, the investment threshold becomes

$$h^*_{OI} = \frac{\psi_1 C_I}{\psi_1 - 1} \frac{\phi(b\sqrt{d_I})}{\phi((b+\sigma)\sqrt{d_I})},$$

where h^*_{OI} represents the optimal investment threshold when divestment is not possible and d_I is the usual investment lag. The lower diagram of Fig. 1 graphically illustrates the divergence between the flexibility premium associated with an irreversible abatement option (θ_{OI}) and a reversible abatement option (θ_D). The intuition behind this result is that without reversibility, the firm must endure the potentially extremely bad market conditions that a high level of uncertainty may generate. As such, the associated flexibility premium must be higher, as illustrated in the lower diagram of Fig. 1.

The results of the quantitative illustration have two kinds of practical relevance. First, they may partly explain the empirical observation that, in most existing cap-and-trade programmes where emissions permits prices are low, all currently adopted abatement technology is mature, ready to implement and easily reversible or even infinitely reversible. Second, they suggest that for industries characterised by long investment lags or irreversible investments in durable equipment, achieving net-zero emissions requires permits to be in the high-price range as advocated by Stern and Stiglitz (2017) and Burke et al. (2019).

5. Conclusion

Under a cap-and-trade programme, regulated firms have two basic alternatives to achieve compliance: adoption of pollution abatement technologies or trading of emissions permits. Firms are indifferent between these compliance strategies when they attain the same total amount of emissions reduction and have the same total cost. Importantly, indifference requires perfect substitution. Trading is reversible and instantaneous, whereas abatement investments are often not reversible and the technology adoption is usually subject to implementation lags. As such, from the perspective of the regulated firms, trading of emissions permits is a more flexible compliance strategy since trades can be easily adjusted to changing market conditions.

The academic literature has long recognised irreversibility, and recently implementation lags, to be part of the explanation for the limited adoption of less mature abatement technologies that require large upfront investments in durable equipment whose implementation/installation can take a very long time. This paper's results make two important contributions to this literature. First, we offer an intuitive decomposition of the permit price, offering a conceptual quantification of the embedded flexibility premium that encompasses both the *reversibility* and the *immediateness* optionalities. Second, we formulate a general investment–divestment problem that appropriately captures varying lengths of implementation delays and the ability to divest, namely investment reversibility, by adopting the Parisian setup used in option pricing. Finally, we illustrate how to quantify the impact of both irreversibility and implementation lags on the price of emissions permits.

Of course, our model has been based on many simplifying assumptions such that our study focusses on only two key considerations: varying lengths of implementation lags and a binary alternative between reversible and irreversible investments. These assumptions rest mainly on the empirical observation that available abatement options largely consist of mature technologies that can be implemented quickly after the adoption decision, or they are irreversible and durable investments.

The policy implications of our theoretical results and quantitative illustration are nevertheless worth discussing. Regulated industries whose investments are characterised by long implementation lags or irreversible durable equipment respond to cap-and-trade regulations differently. The flexibility premium increases with increasing investment–divestment lags and decreasing degrees of reversibility, which means that less stringent cap-and-trade regulations have smaller effects on these abatement technologies. These analytical and quantitative results speak directly to the debate about higher permits prices, advocated by Stern and Stiglitz (2017) and Burke et al. (2019), necessary to achieve zero emissions.

Appendix A. Definitions and results

Taking advantage of some standard results in Revuz and Yor (1991) and Chesney et al. (1997b) and utilising the key results in Costeniuc et al. (2008), the appendix reconstructs the elements required to rewrite the problem (3) in terms of the key parameters that characterise the available abatement technology (irreversibility and implementation lags) and describe the flexibility premium.

We first define the Brownian meander and list some of its properties. In words, the Brownian meander is a piece of Brownian motion that spends all its time away from its starting point. We refer to Revuz and Yor (1991) for complete mathematical details of the Brownian meander. Then, we present the connection between the Brownian meander and the Parisian condition.

Let $(Z_t, t \geq 0)$ be a standard Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. For each $t > 0$, we define the random variables

$$g_t = \sup\{s : s \leq t, Z_s = 0\},$$

$$d_t = \inf\{s : s \geq t, Z_s = 0\}.$$

The interval (g_t, d_t) is called the 'interval of the Brownian excursion' which straddles time t. For instant u in this interval, $sgn(Z_t)$ remains constant. In particular, g_t represents the last time the Brownian motion crossed level 0. Recalling standard results in Revuz and Yor (1991) and Chesney et al. (1997b), g_t is not a stopping time for the Brownian filtration $(\mathcal{F}_t)_{t\geq 0}$, but for the slow Brownian filtration $(\mathcal{G}_t)_{t\geq 0}$, which is defined by $\mathcal{G}_t = \mathcal{F}_{g_t} \vee \sigma(sgn(Z_t))$. The Brownian motion continuously crosses the level 0, thus the slow Brownian filtration is a way of representing the information about (a) the last instant the Brownian motion crossed level 0 and (b) whether it is above or below the level 0 (Revuz and Yor, 1991and Chesney et al., 1997b).

The Brownian meander process ending at t is defined as (see Revuz and Yor, 1991)

$$m_u^{(t)} = \frac{1}{\sqrt{t - g_t}} |Z_{g_t + u(t-g_t)}|, \quad 0 \leq u \leq 1.$$

The process $m_u^{(t)}$ is the non-negative and normalised Brownian excursion which straddles time t and is independent of the σ-field $(\mathcal{G}_t)_{t\geq 0}$. When $u = 1$ and $t = 1$, we conveniently denote $m_1 = m_1^{(1)}$. The random variable m_1 will play a central role in the calculation of many other variables that will be introduced later on. Recalling Revuz and Yor (1991), the distribution of m_1 is

$$\mathbb{P}(m_1 \in dx) = x \exp(-\frac{1}{2}x^2) 1_{x>0} dx,$$

and the moment-generating function $\phi(z)$ is given by (see Revuz and Yor, 1991)

$$\phi(z) = \mathbb{E}(\exp(zm_1)) = \int_0^\infty x \exp(zx - \frac{1}{2}x^2) dx.$$

We follow in the footsteps of Costeniuc et al. (2008), and define the first instant when the Brownian motion spends d units of time consecutively above (or below) level 0. For $d \geq 0$, we define the random variables

$$\begin{aligned} H_d^+ &= \inf\{t \geq 0 : t - g_t \geq d, \quad Z_t \geq 0\} \\ H_d^- &= \inf\{t \geq 0 : t - g_t \geq d, \quad Z_t \leq 0\} \end{aligned} \tag{A.1}$$

The variables H_d^+ and H_d^- are $\mathcal{G}_t$-stopping times and, hence, $\mathcal{F}_t$-stopping times (see Revuz and Yor (1991)for more details). Using Eq. (A.1) we can easily deduce that the pairs of random variables H_d^+ and $Z_{H_d^+}$, and H_d^- and $Z_{H_d^-}$, are independent.

Chesney et al. (1997a) were the first to calculate the Laplace transform of H_d^+. Below we report the main results omitting the proofs. We refer to Chesney et al. (1997a) for a formal and rigorous demonstration of the results.

Laplace transform of H_d^+: Let H_d^+ be the stopping time defined in (A.1) and ϕ the moment-generating function defined in Eq. (A.1). For any $\lambda > 0$,

$$\mathbb{E}[\exp(-\lambda H_d^+)] = \frac{1}{\phi(\sqrt{2\lambda d})}.$$

The proof is based on the Azèma martingale, $\mu_t = sgn(Z_t)\sqrt{t - g_t}$– a remarkable $(\mathcal{G}_t)$ martingale. The same results also hold when H_d^+ is replaced with H_d^-.

So far, we have only looked at the Brownian motion excursions above or below level 0. More generally, we can define for any $a \in \mathbb{R}$ and any continuous stochastic process X that

$$\begin{aligned} g_t^{X_0,a}(X) &= \sup\{s : s \leq t, X_0 = X_0, X_t = a\}, \\ H^+_{(X_0,a),d}(X) &= \inf\{t \geq 0 : t - g_t^{X_0,a} \geq d, X_0 = X_0, \quad X_t \geq a\}, \\ H^-_{(X_0,a),d}(X) &= \inf\{t \geq 0 : t - g_t^{X_0,a} \geq d, X_0 = X_0, \quad X_t \leq a\} \end{aligned} \tag{A.2}$$

Thus, building on the previous definitions, $g_t^{X_0,a}(X)$ represents the last time the process Xcrossed level a. As for the Brownian motion case, $g_t^{X_0,a}(X)$ is not a stopping time for the Brownian filtration $(\mathcal{F}_t)_{t\geq 0}$, but for the slow Brownian filtration $(\mathcal{G}_t)_{t\geq 0}$. The random variables $H^+_{(X_0,a),d}(X)$ (resp. $H^-_{(X_0,a),d}(X)$) represent the first instant when the process X spends d units of time above (or below) the level a. The variables $H^+_{(X_0,a),d}(X)$ and $H^-_{(X_0,a),d}(X)$ are $\mathcal{G}_t$-stopping times and hence $\mathcal{F}_t$-stopping times. In our notation, we indicate the starting point of the process X, the level a and the length of time d.

While the identification of the starting point seems unnecessary, it turns out to be extremely helpful in the context of the Parisian condition. In the context of the abatement investment discussed in the main text, the value of the reversibility/divestment and immediateness optionalities depend on the *excursions* of the underlying investment value above or below a certain barrier. The relevance and effect of a time lag between the instant the decision is taken and the instant the decision is implemented has been explored in the optimal dividend payment literature as well (see Dassios and Wu, 2009 and Cheung and Wong, 2017).

Another relevant random variable is the first hitting time of level a, which we define below:

$$T_{X_0,a}(X) = \inf\{s : X_0 = X_0, X_s = a\}.$$

Appendix B. Parisian condition

According to the notation introduced in Section 3, and considering an underlying process $(V_t, t \geq 0)$ to ease the notation, the investment stopping time τ_I which satisfies the Parisian condition corresponds to $H^+_{(V_0,h_I),d_I}(V)$. In order to express the divestment stopping time τ_D in mathematical formulas, we need to extend the definition of $H^+_{(V_0,h_I),d_I}(V)$. Let τ be any stopping time, $a \in \mathbb{R}$,X a continuous stochastic process and $g_t^{X_0,a}(X)$ as defined in Eq. (A.2). Then

1. the first instant after τ when process X spends d units of time above or below level a is given by the stopping time $H^{+,\tau}_{(X_0,a),d}(X)$ or $H^{-,\tau}_{(X_0,a),d}(X)$, respectively
$$H^{+,\tau}_{(X_0,a),d}(X) = \inf\{t \geq \tau : t - g_t^{X_0,a} \geq d, X_0 = X_0, \quad X_t \geq a\},$$
$$H^{-,\tau}_{(X_0,a),d}(X) = \inf\{t \geq \tau : t - g_t^{X_0,a} \geq d, X_0 = X_0, \quad X_t \leq a\};$$
2. the first hitting time after τ of level a is the stopping time $T^{\tau}_{X_0,a}(X)$
$$T^{\tau}_{X_0,a}(X) = \inf\{s \geq \tau : X_0 = X_0, X_s = a\}.$$

If X has the strong Markov property and τ is a finite stopping time, we have the following equalities in distribution $H^{+,\tau}_{(X_0,a),d}(X) = H^{+}_{(X_\tau,a),d}(X)$,$H^{-,\tau}_{(X_0,a),d}(X) = H^{-}_{(X_\tau,a),d}(X)$, and $T^{\tau}_{X_0,a}(X) = T_{X_\tau,a}(X)$. Using these results, Costeniuc et al. (2008)can state the expression of the stopping times τ_I and τ_D which satisfy the Parisian condition. Below, we reproduce the key expression of interest and refer to Costeniuc et al. (2008) for a detailed derivation of the decomposition of the divestment Parisian time.

Stopping times and the Parisian condition: Let τ_I and τ_D be the stopping times corresponding to the Parisian condition with time windows d_I,d_D and levels h_I,h_D respectively. Then the following equalities hold
$$\tau_I = H^{+}_{(V_0,h_I),d_I}(V), \quad \tau_D = H^{-,\tau_I}_{(V_0,h_D),d_D}(V).$$
Otherwise, in terms of the drifted Brownian motion, the Parisian stopping times are
$$\tau_I = H^{+}_{(V_0,h_I),d_I}(V) = H^{+}_{(l_0,l_I),d_I}(X), \quad \text{where } l_0 = 0, \quad \text{and} \quad l_I = \frac{1}{\sigma}\log\left(\frac{h_I}{V_0}\right),$$
and
$$\tau_D = H^{-,\tau_I}_{(V_0,h_D),d_D}(V) = H^{-,\tau_I}_{(l_0,l_D),d_D}(X), \quad \text{where } l_0 = 0, \quad \text{and} \quad l_D = \frac{1}{\sigma}\log\left(\frac{h_D}{V_0}\right).$$

Decomposition of divestment Parisian time: Let τ be any finite stopping time such that τ and V_τ are independent, and assume $h_D \leq V_\tau$ a.s. Then the following equality in distribution holds
$$H^{-,\tau}_{(V_0,h_D),d_D}(V) = \tau + T_{V_\tau,h_D}(V) + H^{-}_{(h_D,h_D),d_D}(V),$$
and the terms of the sum are independent. A similar relationship holds for $H^{+,\tau}_{(V_0,h_I),d_I}(V)$ if we assume $V_\tau \leq h_I$ a.s.

Appendix C. Reformulation of the investment–divestment problem

To obtain the optimal investment and divestment thresholds in analytic form, Costeniuc et al. (2008) calculate all terms that enter into the maximisation problem. Below we report the key expressions that allow us to rewrite the problem (3) in terms of the key model parameters (the implementation lags and the investment and divestment costs) and the main decision variables (the investment and divestment thresholds).

We first introduce the Parisian investment time,
$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_I}\right] = \left(\frac{V_0}{h_I}\right)^{\frac{\sqrt{2\lambda}}{\sigma}} \frac{1}{\phi(\sqrt{2\lambda d_I})},$$
the moment-generating function for the process X_t defined in (5), stopped at the Parisian investment time
$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda X_{\tau_I}}\right] = \left(\frac{h_I}{V_0}\right)^{-\frac{\lambda}{\sigma}} \phi(-\lambda\sqrt{d_I}),$$
and the first hitting time of X starting at the Parisian investment time
$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda T_{(X_{\tau_I},l_D)}(X)}\right] = \left(\frac{h_D}{h_I}\right)^{\frac{\sqrt{2\lambda}}{\sigma}} \phi(-\sqrt{2\lambda d_I}).$$
We now consider the divestment action and introduce the Parisian divestment time
$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_D}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_I}\right] \frac{\phi(-\sqrt{2\lambda d_I})}{\phi(\sqrt{2\lambda d_D})} \left(\frac{h_D}{h_I}\right)^{\frac{\sqrt{2\lambda}}{\sigma}},$$

the moment-generating function for the process X_t defined in (5), stopped at the Parisian divestment time

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda X_{\tau_D}}\right] = \left(\frac{h_D}{V_0}\right)^{-\frac{\lambda}{\sigma}} \phi(\lambda\sqrt{d_D}).$$

Equipped with these expressions, we can rewrite the first term appearing in the investment–divestment problem (3) as:

$$\mathbb{E}_{\mathbb{P}}\left[e^{-\rho\tau_I}(V_{\tau_I} - C_I)1_{\{\tau_I<\infty\}}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-(\rho+\frac{b^2}{2})\tau_I}\right]\left(\frac{h_I}{V_0}\right)^{\frac{b}{\sigma}} \phi(b\sqrt{d_I})\times$$

$$\times\left\{h_I\frac{\phi(\sqrt{d_I}(\sigma+b))}{\phi(b\sqrt{d_I})} - C_I\right\}.$$

Similarly, we can rewrite the second term in (3) as:

$$\mathbb{E}_{\mathbb{P}}\left[e^{-\rho\tau_D}(C_D - V_{\tau_D})1_{\{\tau_D<\infty\}}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-(\rho+\frac{b^2}{2})\tau_I}\right]\left(\frac{h_I}{V_0}\right)^{\frac{b}{\sigma}}\left(\frac{h_I}{h_D}\right)^{\frac{-b-\sqrt{2\rho+b^2}}{\sigma}}\times$$

$$\times\frac{\phi(-\sqrt{(2\rho+b^2)d_I})}{\phi(\sqrt{(2\rho+b^2)d_D})}\phi(-b\sqrt{d_D})\left\{C_D - h_D\frac{\phi(-(b+\sigma)\sqrt{d_D})}{\phi(-b\sqrt{d_D})}\right\}.$$

References

Aguerrevere, F.L., 2015. Equilibrium Investment Strategies and Output Price Behavior: A Real-Options Approach. The Review of Financial Studies 16 (4), 1239–1272.
de Almeida, G.P., Zemsky, P., 2003. The Effect of Time-to-Build on Strategic Investment under Uncertainty. The RAND Journal of Economics 34 (1), 166–182.
Bar-Ilan, A., Strange, C., 1996. Investment lags.. The American Economic Review 86, 610–622.
Bloom, N., 2014. Fluctuations in Uncertainty. Journal of Economic Perspectives 28 (2), 153–176.
Brekke, K.A., Øksendal, B., 1994. Optimal switching in an economic activity under uncertainty.. SIAM Journal on Control and Optimization 32.
Brennan, M.J., Schwartz, E.S., 1985. Evaluating natural resource investments.. Journal of Business 58.
Bultan, L.T., 2005. Real options, irreversible investment and firm uncertainty: New evidence from U.S. firms. Review of Financial Economics 14 (3), 255–279.
Burke, J., Byrnes, R., Fankhauser, S., 2019. How to price carbon to reach net-zero emissions in the UK. Technical Report. Grantham Research Institute Policy Brief.
Calel, R., 2020. Adopt or Innovate: Understanding Technological Responses to Cap-and-Trade. American Economic Journal: Economic Policy 12 (3), 170–201.
Carmona, R., Fehr, M., Hinz, J., Porchet, A., 2009. Market design for emission trading schemes. SIAM Review 9 (3), 465–469.
Chao, H., Wilson, R., 1993. Option value of emission allowances.. Journal of Regulatory Economics 5, 233–249.
Chesney, M., Jeanblanc, M., Yor, M., 1997. Brownian excursions and parisian barrier options.. Advances in Applied Probability 29, 165–184.
Chesney, M., Jeanblanc, M., Yor, M., 1997. Parisian options and excursion theory.. Advances in Applied Probability 29, 165–184.
Cheung, E.C., Wong, J., 2017. On the dual risk model with Parisian implementation delays in dividend payments. European Journal of Operational Research 257 (1), 159–173.
Cicala, S., 2015. When Does Regulation Distort Costs? Lessons from Fuel Procurement in US Electricity Generation. American Economic Review 105 (1), 411–444.
Costeniuc, M., Schnetzer, M., Taschini, L., 2008. Entry and exit decision problem with implementation delay. Journal of Applied Probability 45 (4), 1039–1059.
Cronshaw, M.B., Kruse, J.B., 1996. Regulated firms in pollution permit markets with banking.. Journal of Regulatory Economics 9, 179–189.
Dassios, A., Wu, S., 2009. On barrier strategy dividends with Parisian implementation delay for classical surplus processes. Insurance: Mathematics and Economics 45 (2), 195–202.
Delarue, E., Ellerman, A.D., D'Haeseleer, W., 2010. Short Term CO_2 Abatement in the European Power Sector: 2005-2006. Climate Change Economics 1 (2), 113–133.
Delarue, E., Voorspools, K., D'Haeseleer, W., 2008. Fuel Switchinig in the Electricity Sector under the EU ETS: Review and Prospective. Journal of Energy Engineering 134 (2), 40–46.
Dixit, A., 1989. Entry and exit decisions under uncertainty.. Journal of Political Economy 97.
Dixit, A.K., Pindyck, R.S., 1994. Investment under Uncertainty.. Princeton University Press, Princeton, New Jersey.
European Commission, 2005. EU action against climate change: EU emissions trading - an open scheme promoting global innovation. Technical Report. European Commission.
Fowlie, M., Holland, S.P., Mansur, E.T., 2012. What do Emissions Markets Deliver and to Whom? Evidence from the Southern California's NOx Trading Program. American Economic Review 102 (2), 965–993.
Goulder, L., 2013. Markets for Pollution Allowances: What Are the (New) Lessons? Journal of Economic Perspectives 27 (1), 87–102.
Hitzemann, S., Uhrig-Homburg, M., 2018. Equilibrium Price Dynamics of Emission Permits. Journal of Financial and Quantitative Analysis 53 (4), 1653–1678.
ICAP, 2020. Emissions Trading Worldwide: Status Report 2020. Technical Report. International Carbon Action Partnership.
Kalouptsidi, M., 2014. Time to build and fluctuations in bulk shipping. American Economic Review 104 (2), 564–608.
Kerr, S., Newell, R., 2003. Policy-Induced Technology Adoption: Evidence from the US LEad Phasedown. Journal of Industrial Economics 51, 317–343.
Kollenberg, S., Taschini, L., 2016. Emissions trading systems with cap adjustments. Journal of Environmental Economics and Management 80 (1), 20–36.
Lintunen, J., Kuusela, O., 2018. Business cycles and emission trading with banking. European Economic Review 101 (1), 397–417.
Majd, S., Pindyck, R.S., 1987. Time to build, option value, and investment decisions.. Journal of Financial Economics 18, 7–27.
McDonald, R., Siegel, D., 1986. The value of waiting to invest.. The Quarterly Journal of Economics 101 (4), 707–728.
Montgomery, W., 1972. Markets in licenses and efficient pollution control programs. Journal of Economic Theory 5.
Oh, H., Yoon, C., 2020. Time to build and the real-options channel of residential investment. Journal of Financial Economics 135 (1), 255–269.
Øksendal, B., 2005. Optimal stopping with delay information.Working paper series, Department of Mathematics, University of Oslo.
Paciorek, A., 2013. Supply constraints and housing market dynamics. Journal of Urban Economics 77 (1), 11–26.
Pindyck, R.S., 1988. Irreversible investment, capacity choice, and the value of the firm.. American Economic Review 78, 969–985.
Revuz, D., Yor, M., 1991. Continuous Martingales and Brownian Motion.. Springer-Verlag, Berlin Heidelberg New York.
Rockstróm, J., Gaffney, O., Rogelj, J., Meinshausen, M., Nakicenovic, M., Schellnhuber, H., 2017. A roadmap for rapid decarbonization. Science 355 (6331), 1269–1271.
Rubin, J.D., 1996. A model of intertemporal emission trading, banking, and borrowing.. Journal of Environmental Economics and Management 31, 269–286.
Schennach, S.M., 2000. The economics of pollution permit banking in the context of Title IV of the 1990 Clean Air Act Amendments.. Journal of Environmental Economics and Management 40, 189–210.

Schmalensee, R., Joskow, P., Ellerman, A.D., Montero, J.P., Bailey, E.M., 1998. An Interim Evaluation of Sulfur Dioxide Emissions Trading. The Journal of Economic Perspectives 12 (3), 53–68.
Schmalensee, R., Stavins, R., 2017. The design of environmental markets: What have we learned from experience with cap and trade? Oxford Review of Economic Policy 33 (4), 572–588.
Sødal, S., 2006. Entry and exit decisions based on a discount factor approach. Journal of Economic Dynamics & Control 30 (1), 1963–1986.
Sødal, S., Koekebakker, S., Aadland, R., 2008. Market switching in shipping — A real option model applied to the valuation of combination carriers. Review of Financial Economics 17 (3), 183–203.
Stern, N., Stiglitz, J., 2017. Report of the high-level commission on carbon prices.. Technical Report. World Bank, Washington D.C..
Stranlund, J., Murphy, J., Spraggon, J., 2011. An experimental analysis of compliance in dynamic emissions markets. Journal of Environmental Economics and Management 63 (3), 414–429.
Zhao, J., 2003. Irreversible abatement investment under cost uncertainties: Tradable emission permits and emissions charges.. Journal of Public Economics 87, 2765–2789.
Zhou, C., 2000. Time-to-Build and Investment. Review of Economics and Statistics 82 (1), 273–282.